# Neural Networks Approach to the Determination of the Machining Parameters

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A neural networks based approach to determine the appropriate machining parameters such as speed, depth of cut and feed is proposed in this study. In this approach neural networks were used for building automatic process planning systems. Training of neural networks was performed with back propagation method by using data sets sampled in a staindard handbook. These networks consist of simple processing elements or nodes capable of processing information in response to external inputs. This approach saves computing time and storage space. In addition, it provides easy extendability as new data become available. Currently, the system provides three neural networks: for turning, for milling and for drilling operations. The performance of the trained neural network for drilling is evaluated to examine how well it predicts the machining parameters. Test results show that the neural network for the turning operation is able to predict the machining parameter values within an acceptable error rate.

Key Words: Neural Networks, Back Propagation Model, Machining Parameters, Automatic Process Planning

## 1. Introduction

In machining operations, the surface finish, force, and power consumption are directly affected by the machining parameters, feed rate, speed, and depth of cut. The selection of the machining parameters affects the quality, time and cost to produce a part. These parameters are neither arbitrary nor constants for different machining processes.

Earlier studies on economical selection of machining parameters are grouped into three categories: optimal techniques, handbook databases, and machinist experience. In optimal techniques, main objective functions proposed for the machining operations are related to the production cost (Bhattacharyya, 1970), production rate (Field, 1969), and profit rate (Wu, 1966). The proposed constraints are subject to machine tool capacity and component quality specifications. Various methods have been applied to maximize or minimize an objective function under constraints, including the mathematical programming methods (Barrow, 1971, Yellowley, 1989, Ermer, 1971, Iwata, 1972). These optimal technique approach requires quantifying the result of using a set of parameters in the form of mathematical relationships that can be optimized for a desired result. Due to the complexity of interrelationships between the parameters, such techniques have not widely used in practice.

Machinists can select a good set of parameters for a required outcome through experience, However, experience varies widely and is very difficult to capture in a machine useable format. Handbooks of test results contain recommended machining parameters for efficient machining. The data set contained in these handbooks is very large. While it may be possible to house it in a database, and use search engines for parameter determination, such an approach would be wasteful.

Neural networks have been implemented and used in this work for dealing with the difficulty in

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choosing recommended machining parameters. These networks consist of simple processing elements or nodes capable of processing information in response to external inputs. This approach saves computing time and storage space. In addition, it provides easy extendability as new data become available.

# 2. Basic Machining Parameters

## 2.1 Feed and feed rate

Feed F can be defined as the relative lateral movement between the tool and the workpiece during a machining operation. It corresponds to the thickness of the chip produced by the operation. In turning and drilling operations, it is defined as the advancement of the cutter per revolution of the workpiece(turning) and tool (drilling). The unit is mpr(millimeter per revolution). In milling, it is defined as the advancement of the cutter per cutter-tooth revolution. The unit is millimeter per revolution per tooth.

Feed rate  $V_f$  is defined as the speed of feed. The unit is mpm(millimeter per minute). Mathematically feed rate can be expressed as follows:

$$V_f = FnN$$

where, n= the number of teeth in the cutter for milling, n=1 for drilling and turning

N=rotation speed of the cutter(drilling and milling) or workpiece(turning) in rpm

## 2.2 Cutting speed

The cutting speed V can be defined as the maximum linear speed between the tool and the workpiece. The cutting speed is determined as a function of the diameter of the workpiece or the tool, and rotation speed. It can be represented as follows:

$$V = \frac{\pi DN}{1000}$$

where, V = speed, meter per minute D = diameter, millimeters N = rotation speed, rpm

## 2.3 Depth of cut

The depth of cut is determined by the width of

the chip. During the roughing operation, the depth of cut is usually much greater than for the finishing operation. For turning, it is one-half the difference between the inner and outer diameters of the workpiece. Mathematically it can be obtained as follows:

$$d_p = \frac{D_o - D_o}{2}$$

where,  $d_p$ =depth of cut, millimeters  $D_o$ =outer diameter, millimeters  $D_i$ =inner diameter, millimeters

# 3. Neural Network Model for Machining Parameters

The optimal machining parameters (depth of cut, speed, and feed rate) are calculated using Back Propagation Neural Networks (BPN). The neural network sub module is available for use by a cell supervisor when new data related to machining parameters become available. Currently, the system provides three neural networks: for turning, for milling, and for drilling operations. These networks have been trained based on standard handbook data (Machinability Data Center, 1972).



Fig. 1 Structure of BPN model

The multi-layered feed forward BPN, with full interconnection, consists of an output layer, input layer, and one or more hidden layers. Each layer has a set number of nodes that are chosen to fit the problem at hand(see Fig. 1). Each node (processing element) is a neuron that processes a set of inputs applied either from outside( $X_i$ ) or from a previous layer( $H_z$ ) to produce the output by applying an activation function, f, to the weighted sum of inputs. The actual output,  $Y_i$ , of the BPN with two hidden layers is given by the following form.

$$Y_l = f\left(y_l + \sum_{k} W_{lk} P_k\right) \tag{1}$$

where,

$$P_{k} = f\left(p_{k} + \sum_{j} W_{kj}H_{j}\right)$$
$$H_{j} = f\left(h_{j} + \sum_{i} W_{ji}X_{i}\right)$$

The subscripts *i*, *j*, *k*, *l* represent the number of nodes in the input, hidden, and output layers, respectively. The biases  $(h_z, p_k, y_l)$  and connection weights (W) are determined in the same way during the learning of the network with the preselected training input data set. BPN learning characteristics can be improved by adjusting the dynamic range of neuron output within  $\pm$  0.5 (Wasserman, 1989). The improved activation function is given by the following form.

$$Y = f(NET) = -0.5 + \frac{1}{1 + e^{-NET}}$$
(2)

and the derivative of Y is given by

$$f(NET) = \left(\frac{1}{2} + Y\right)\left(\frac{1}{2} - Y\right) \tag{3}$$

The back propagation learning algorithm is a generalized delta rule in which the connection weights, W(m) at m-th iteration time, is updated with the general second order linear stochastic difference equation based on the gradient descent method (Kosko, 1991).

$$W(m+1) = W(m) + \eta \Delta W(m) + \alpha \Delta W(m-1)$$
(4)

The  $\eta$  is the learning rate and  $\alpha$  is the momentum term to be chosen for guaranteed and fast convergence. W(m) is obtained by minimizing the summed squared error  $E_m$  with respect to W:

$$\Delta W(m) = -\frac{\partial E_m}{\partial W}$$
(5)

where,

$$E_m = \frac{1}{2} \sum_{p} (T_p - Y_p^m)^2$$

and  $T_p$  is the target output for training input pattern p, and  $Y_p^m$  is the trained output for training input pattern p at i-th iteration time.

Using the chain rule, Eq. (5) renders the following for node 1 in the output layer:

$$\Delta W_{lk}(m) = -\frac{\partial E_m}{\partial Y_l^m} \frac{\partial Y_l^m}{\partial W_{lk}^m}$$
  
$$= -\frac{\partial E_m}{E Y_l^m} f(Y_l^m) P_k^m$$
  
$$= [T_l - Y_l^m] [\frac{1}{2} + Y_l^m]$$
  
$$[\frac{1}{2} - Y_l^m] P_k^m \qquad (6)$$

Similarly, for the hidden layers Eq. (5) yields:

$$\begin{split} \mathcal{\Delta}W_{kj}(m) &= -\frac{\partial E_m}{\partial P_k^m} \frac{\partial P_k^m}{\partial W_{kj}^m} \\ &= -\left[\sum_l \frac{\partial E_m}{Y_l^m} \frac{\partial Y_l^m}{\partial P_k^m}\right] f(P_k^m) H_j^m \\ &= (\sum_l [T_l - Y_l^m] f(Y_l^m) W_{lk}^m) \\ f(P_k^m) H_j^m \\ &= (\sum_l [T_l - Y_l^m] \\ \left[\frac{1}{2} + Y_l^m\right] \left[\frac{1}{2} - Y_l^m\right] \cdot W_{lk}^m) \\ \left[\frac{1}{2} + P_k^m\right] \left[\frac{1}{2} - P_k^m\right] H_j^m \quad (7) \\ \mathcal{\Delta}W_{ji}(m) &= -\frac{\partial E_m}{\partial H_j^m} \frac{\partial H_j^m}{\partial W_{ji}^m} \\ &= -\left(\sum_l \sum_k \frac{\partial E_m}{Y_l^m} \frac{\partial Y_l^m}{\partial P_k^m} \frac{\partial P_k^m}{\partial H_j^m}\right) \\ f(H_j^m) X_i \\ &= (\sum_l \sum_k [T_l - Y_l^m] f(Y_l^m) W_{lk}^m) \\ f(P_k^m) W_{kj}^m) f(H_j^m) X_i \\ &= (\sum_l \sum_k [T_l - Y_l^m] \\ \left[\frac{1}{2} + Y_l^m\right] \left[\frac{1}{2} - P_k^m\right] W_{kj}^m) \\ &\left[\frac{1}{2} + P_k^m\right] \left[\frac{1}{2} - H_j^m\right] X_i \quad (8) \end{split}$$

A four layered BPN is used for obtaining machining parameters. The output layer has three (or two) nodes representing parameters (i. e., depth of cut, speed, feed) for the given machine operation parameters, in this case tool material, work piece material, geometry, etc., which specifically constitute the number of nodes in the input layer.

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Input vector		Input	Referenced output variables			
	Material	Hardness (BHN)	Hole diameter (mm)	Tool material	Speed (mpm)	Feed (mpr)
1	1212	125	12.7	M10	38.1	0.254
2	1213	175	25.4	M10	38.1	0.457
3	1212	125	50.8	M10	38.1	0.635
4	1215	125	19.05	<b>M</b> 7	39.6	0.381
5	1108	175	25.4	M1	36.6	0.457
6	11L13	225	38.1	M10	41.2	0.508
7	11L14	125	19.05	M10	39.6	0.381
8	12L13	125	50.8	<b>M</b> 7	30.5	0.635
9	1005	150	6.35	M1	27.4	0.127
10	1012	150	38.1	M1	27.4	0.457
11	1019	200	12.7	<b>M</b> 7	24.4	0.229
12	1021	200	38.1	<b>M</b> 10	19.8	0.381
13	1030	200	50.8	M10	22.9	0.559
14	1040	250	19.05	<b>M</b> 7	18.3	0.254
15	1049	300	25.4	<b>M</b> 33	15.2	0.305

 Table 1 Training input data vectors for drilling

## 4. Training Data Generation

The procedure for generating the randomized training input data consists of two steps. In the first step, the lower and the upper bounds on the input variables are defined. Within these limits, the sample set is randomly obtained. These limits are determined by considering the reasonable range of the values for each variable that is likely to occur in the operation of a particular cell based on its constituent machines. For the case of drilling in the demonstration cell, the range of parameter hardness is between 125 BHN and 535 BHN.

In the second step, an input data set of size n from k input variables is formed. As an example, there are four inputs for drilling. These are the raw material, the hardness, the tool material, and the hole diameter. Regarding the number of training input data needed for generalization, Ahmad (1989) suggests that the input size n in the order of  $k^{2/3}$  random patterns is sufficient for a network to learn with high degree of generality. In the drilling case, a sample size of n=90 is used. Some

of the data set used for training the BPN is shown in Table 1.

## 5. Network Learning Result

The network learning is performed with the training data to find out the weight matrix and node biases. In order to cope with the long training time of BPN, some measures for improving efficiency are taken. The input patterns and target outputs are normalized and scaled within the range -0.5 and 0.5. The learning rate  $(\eta)$  and the momentum  $(\alpha)$  are adjusted during the training process for speeding up the convergence. Typical values of learning rate are 1.0 to 3.0 for the small training data set size, and 0.1 to 0.3 for the large training data set size. In the case of drilling, a learning rate of 0.2 was used. The values of momentum are kept less than 1.0. The termination criterion is set for a maximum output error of  $\varepsilon = 0.01$ . An example of the resultant weight matrix and the biases for the case of drilling is shown in Table 2-

NC	ode	$W_{ji}$ (i:Preceeding)								
i	j	1	2	3	4	5	6	7		
1		-0.63548	-6.88472	-2.96061	2.91050	2,50716	-0.52509	1.23056		
2		8.11659	13.75063	-1.47320	1.66996	-8.75082	5.73054	-9.90321		
3		1.61627	-6.37272	0.72570	-5.03440	3.53972	-2.83954	9.18652		
4		0.72026	0.15873	-2.38315	3.18416	-0.47322	-5.98722	3.05997		
1		1.37453	-3.63443	-8.86177	-8.96499	-4.26739	-4.20179	-2.45929		
2		1.30533	4.91780	0.90458	-0.32502	-0.06969	-0.00814	2.36585		
3		0.84227	-3.95956	3.87847	-10.10952	3.39331	17.23709	1.15399		
4		0.48097	1.06324	-12.29541	-0.35674	-2.41680	1.14906	5.16952		
5		2.22554	-1.60456	3.73718	-3.28816	0.07215	0.88020	0.10161		
6		-2.56526	-2.19187	-3.09466	3.00954	-1.94243	-3.62429	1.41826		
7		-3.48167	0.55946	-0.00896	2.49217	-2.23343	4.67254	1.94875		
1		-2.24265	4.05617	-4.74141	1.09121	-3.03116	-5.29675	-5.17504		
2		9.27810	-4.81895	-0.18701	-9.79425	2.15580	-3.08881	-3.76751		
		I					· · · · · · · · · · · · · · · · · · ·			
	i 1 2 3 4 1 2 3 4 5 6 7 1 2	i j 1 2 3 4 1 2 3 4 5 6 7 1 2	i         j         1           1         -0.63548         2         8.11659           3         1.61627         4         0.72026           1         1.37453         2         1.30533           2         1.30533         3         0.84227           4         0.48097         5         2.22554           6         -2.56526         7         -3.48167           1         -2.24265         2         9.27810	ij121 $-0.63548$ $-6.88472$ 2 $8.11659$ $13.75063$ 3 $1.61627$ $-6.37272$ 4 $0.72026$ $0.15873$ 1 $1.37453$ $-3.63443$ 2 $1.30533$ $4.91780$ 3 $0.84227$ $-3.95956$ 4 $0.48097$ $1.06324$ 5 $2.22554$ $-1.60456$ 6 $-2.56526$ $-2.19187$ 7 $-3.48167$ $0.55946$ 1 $-2.24265$ $4.05617$ 2 $9.27810$ $-4.81895$	ij1231 $-0.63548$ $-6.88472$ $-2.96061$ 2 $8.11659$ $13.75063$ $-1.47320$ 3 $1.61627$ $-6.37272$ $0.72570$ 4 $0.72026$ $0.15873$ $-2.38315$ 1 $1.37453$ $-3.63443$ $-8.86177$ 2 $1.30533$ $4.91780$ $0.90458$ 3 $0.84227$ $-3.95956$ $3.87847$ 4 $0.48097$ $1.06324$ $-12.29541$ 5 $2.22554$ $-1.60456$ $3.73718$ 6 $-2.56526$ $-2.19187$ $-3.09466$ 7 $-3.48167$ $0.55946$ $-0.00896$ 1 $-2.24265$ $4.05617$ $-4.74141$ 2 $9.27810$ $-4.81895$ $-0.18701$	ij12341 $-0.63548$ $-6.88472$ $-2.96061$ $2.91050$ 2 $8.11659$ $13.75063$ $-1.47320$ $1.66996$ 3 $1.61627$ $-6.37272$ $0.72570$ $-5.03440$ 4 $0.72026$ $0.15873$ $-2.38315$ $3.18416$ 1 $1.37453$ $-3.63443$ $-8.86177$ $-8.96499$ 2 $1.30533$ $4.91780$ $0.90458$ $-0.32502$ 3 $0.84227$ $-3.95956$ $3.87847$ $-10.10952$ 4 $0.48097$ $1.06324$ $-12.29541$ $-0.35674$ 5 $2.22554$ $-1.60456$ $3.73718$ $-3.28816$ 6 $-2.56526$ $-2.19187$ $-3.09466$ $3.00954$ 7 $-3.48167$ $0.55946$ $-0.00896$ $2.49217$ 1 $-2.24265$ $4.05617$ $-4.74141$ $1.09121$ 2 $9.27810$ $-4.81895$ $-0.18701$ $-9.79425$	ij123451 $-0.63548$ $-6.88472$ $-2.96061$ $2.91050$ $2.50716$ 2 $8.11659$ $13.75063$ $-1.47320$ $1.66996$ $-8.75082$ 3 $1.61627$ $-6.37272$ $0.72570$ $-5.03440$ $3.53972$ 4 $0.72026$ $0.15873$ $-2.38315$ $3.18416$ $-0.47322$ 1 $1.37453$ $-3.63443$ $-8.86177$ $-8.96499$ $-4.26739$ 2 $1.30533$ $4.91780$ $0.90458$ $-0.32502$ $-0.06969$ 3 $0.84227$ $-3.95956$ $3.87847$ $-10.10952$ $3.39331$ 4 $0.48097$ $1.06324$ $-12.29541$ $-0.35674$ $-2.41680$ 5 $2.22554$ $-1.60456$ $3.73718$ $-3.28816$ $0.07215$ 6 $-2.56526$ $-2.19187$ $-3.09466$ $3.00954$ $-1.94243$ 7 $-3.48167$ $0.55946$ $-0.00896$ $2.49217$ $-2.23343$ 1 $-2.24265$ $4.05617$ $-4.74141$ $1.09121$ $-3.03116$ 2 $9.27810$ $-4.81895$ $-0.18701$ $-9.79425$ $2.15580$	ij1234561 $-0.63548$ $-6.88472$ $-2.96061$ $2.91050$ $2.50716$ $-0.52509$ 2 $8.11659$ $13.75063$ $-1.47320$ $1.66996$ $-8.75082$ $5.73054$ 3 $1.61627$ $-6.37272$ $0.72570$ $-5.03440$ $3.53972$ $-2.83954$ 4 $0.72026$ $0.15873$ $-2.38315$ $3.18416$ $-0.47322$ $-5.98722$ 1 $1.37453$ $-3.63443$ $-8.86177$ $-8.96499$ $-4.26739$ $-4.20179$ 2 $1.30533$ $4.91780$ $0.90458$ $-0.32502$ $-0.06969$ $-0.00814$ 3 $0.84227$ $-3.95956$ $3.87847$ $-10.10952$ $3.39331$ $17.23709$ 4 $0.48097$ $1.06324$ $-12.29541$ $-0.35674$ $-2.41680$ $1.14906$ 5 $2.22554$ $-1.60456$ $3.73718$ $-3.28816$ $0.07215$ $0.88020$ 6 $-2.56526$ $-2.19187$ $-3.09466$ $3.00954$ $-1.94243$ $-3.62429$ 7 $-3.48167$ $0.55946$ $-0.00896$ $2.49217$ $-2.23343$ $4.67254$ 1 $-2.24265$ $4.05617$ $-4.74141$ $1.09121$ $-3.03116$ $-5.29675$ 2 $9.27810$ $-4.81895$ $-0.18701$ $-9.79425$ $2.15580$ $-3.08881$		

Table 2 Weight matrix and biases for drill operation

Diases							
Node Layer	1	2	3	4	5	6	7
Hidden 1	0.75763	-3.67060	-5.54493	-1.96854	0.11851	1.41229	-2.18840
Hidden 2	1.69916	1.13007	3.45615	-0.69308	-0.42873	-2.34371	-1.92592
Output	-0.59779	-7.03206					



Weight matrix

Fig. 2 Comparison between the actual set data and the back-propagation results for the parameter speed

Feed Error Rate



Fig. 3 Comparison between the actual set data and the back-propagation results for the parameter feed

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Input vector	Referenced output variables ( $V_{ref}$ )		Networ variabl	k output les ( $V_{mn}$ )	Ratio (V <sub>nn</sub> /V <sub>ref</sub> )	
	Speed (mpm)	Feed (mpr)	Speed (mpm)	Feed (mpr)	Speed	Feed
1	38.1	0.254	40.00	0.29210	1.05	1.15
2	38.1	0.457	38.10	0.42418	1.00	0.93
3	38.1	0.635	41.15	0.69850	1.08	1.10
4	39.6	0.381	42.18	0.43430	1.04	1.14
5	36.6	0.457	38.68	0.38938	1.06	0.87
6	41.2	0.508	39.23	0.53340	0.95	1.05
7	39.6	0.381	38.81	0.40001	0.98	1.05
8	30.5	0.635	30.20	0.65450	0.99	1.03
9	27.4	0.127	26.86	0.14158	0.98	1.11
10	27.4	0.457	26.86	0.45720	0.98	1.00
11	24.4	0.229	21.95	0.19525	0.90	0.85
12	19.8	0.381	22.13	0.42797	1.12	1.12
13	22.9	0.559	20.96	0.508	0.92	0.91
14	18.3	0.254	17.95	0.22906	0.98	0.90
15	15.2	0.305	13.38	0.32842	0.88	1.08

Table 3 Machining parameters responses from the network on the trained data set

The performance of the trained Neural Network for drilling is evaluated to examine how well it predicts the machining parameters. The test data set and the resulting output machining parameter's output values  $V_{mm}$  from the network are compared to the reference values  $V_{ref}$  which are used for training as shown in Table 3.

The comparison between the actual test set data which are not chosen for training and the back -propagation results should be necessary to determine the possibility of utilizing these networks to the real situation. The ratios between the output values from the network and reference values, i. e., Vnn/Vref, for the speed and the feed are measured and plotted as shown in Figs. 2 and 3, respectively. Two input variables, the tool material and hardness, are set as tool material group M1 M7 M10, and 125 BHN, respectively. The comparison procedure is performed by considering several workpiece materials at each hole diameter to be operated. As shown in Figs, six types of workpice materials considered here are illustrated in legend boxes. Comparison results

show that the Neural Network is able to predict the machining parameter values within 15%.

## 6. Conclusions

A neural network based approach, Back Propagation Neural networks (BPN), to cutting parameter selection for planning machining operations has been described in this paper. The multi-layered feed forward BPNs developed consist of an input layer which allows data to be presented to the network, output layer which holds the response of the network to a given input, and two hidden layers. Each layer has a set number of nodes that are chosen to fit the problem at hand. The average sum of squared error in predicting the test set data was introduced as a means of choosing the optimum node number of hidden layers. For the drilling operation case, the input layer, output layer, first hidden layer, and second hidden layer have 4, 2, 15, and 15 nodes, respectively.

Currently, the system provides three neural

networks: for turning, for milling, and for drilling operations. These networks have been trained based on standard handbook data. Test results have proven that neural networks do in fact have the ability to solve this problem within 15% error rate. This approach saves computing and storage space. In addition, it provides easy extendibility as new data become available.

For the future work, this model combining with fuzzy logic can be extended to adaptive control of machining operations for on-line adjustment of cutting parameters based on information from sensors.

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